# Bridging Pack for Maths & Further Maths

## So, you are considering A Level Mathematics?

This pack contains a programme of activities that will help you in your transition on to the A Level course in September. It is aimed to be used after you have completed your GCSEs over the summer and the contents will be tested in September to check your suitability for the course you have chosen. You need to ensure that you are comfortable with all the skills in this work pack.

## Why is there such a gap between GCSEs and A Levels?

A Levels are – as their name suggests – advanced qualifications, and so require much more of you as a student. They require you to gain a deeper understanding of the subjects you choose to study. There is a lot more independent work required to demonstrate this deeper learning.

*"GCSE is often highly structured with very specific requirements for homework, whereas at A Level there is greater expectation for taking the initiative in going beyond the set reading and utilising the library to read around and consolidate." Dr Ellerby*

# Equipment & Organisation

## Chapter 1: EXPANDING BRACKETS

**Example 1:**

$3(x + 2y) = 3x + 6y$

**Example 2:**

$(x+1)(x+2) = x^2 + 3x + 2$

**Exercise A** Multiply out the following brackets and simplify.

1. $7(4x + 5)$
2. $-3(5x - 7)$
3. $5a - 4(3a - 1)$
4. $4y + y(2 + 3y)$
5. $-3x - (x + 4)$
6. $5(2x - 1) - (3x - 4)$
7. $(x + 2)(x + 3)$
8. $(t - 5)(t - 2)$
9. $(2x + 3y)(3x - 4y)$
10. $4(x - 2)(x + 3)$
11. $(2y - 1)(2y + 1)$
12. $(3 + 5x)(4 - x)$

| **Perfect Square:** | **Difference of two squares:** |
|---|---|
| $(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$ | $(x - a)(x + a) = x^2 - a^2$ |
| $(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$ | $(x - 3)(x + 3) = x^2 - 9$ |

**Exercise B** Multiply out

1. $(x - 1)^2$
2. $(3x + 5)^2$
3. $(7x - 2)^2$
4. $(x + 2)(x - 2)$
5. $(3x + 1)(3x - 1)$
6. $(5y - 3)(5y + 3)$

## Chapter 2: LINEAR EQUATIONS

**Example 1**: Solve the equation $64 - 3x = 25$

**Solution**: There are various ways to solve this equation. One approach is as follows:

Step 1: Add $3x$ to both sides (so that the $x$ term is positive): $64 = 3x + 25$

Step 2: Subtract 25 from both sides: $39 = 3x$

Step 3: Divide both sides by 3: $13 = x$

**Example 2**: Solve the equation $6x + 7 = 5 - 2x$.

**Solution:**

Step 1: Begin by adding $2x$ to both sides $8x + 7 = 5$

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -\frac{1}{4}$

**Exercise A**: Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$
2) $5x - 2 = 13$
3) $11 - 4x = 5$
4) $5 - 7x = -9$
5) $11 + 3x = 8 - 2x$
6) $7x + 2 = 4x - 5$

**Example 3**: Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$

Step 2: Simplify the right hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4: $9x = 18$

Step 5: Divide by 9: $x = 2$

**Exercise B:** Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

**EQUATIONS CONTAINING FRACTIONS**

**Example 4**: Solve the equation $\frac{y}{2} + 5 = 11$

**Solution**:

Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10: $y = 12$

**Example 5**: Solve the equation $\frac{x+1}{4} + \frac{x+2}{5} = 2$

**Solution**:

Step 1: Find the lowest common denominator

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4} + \frac{20(x+2)}{5} = 40$

Step 3: Simplify the left hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}} + \frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}} = 40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13: $9x = 27$

Step 7: Divide by 9: $x = 3$

**Exercise C**: Solve these equations

1) $\frac{1}{2}(x+3) = 5$

3) $\frac{y}{4} + 3 = 5 - \frac{y}{3}$

5) $\frac{7x-1}{2} = 13 - x$

2) $\frac{2x}{3} - 1 = \frac{x}{3} + 4$

4) $\frac{x-2}{7} = 2 + \frac{3-x}{14}$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

**FORMING EQUATIONS**

**Example 8**: Find three consecutive numbers so that their sum is 96.

**Solution**: Let the first number be *n*, then the second is *n* + 1 and the third is *n* + 2.

Therefore $n + (n + 1) + (n + 2) = 96$

$3n + 3 = 96$

$n = 31$

So the numbers are 31, 32 and 33.

**Exercise D:**

1) Find 3 consecutive even numbers so that their sum is 108.

2) The perimeter of a rectangle is 79 cm. One side is three times the length of the other. Form an equation and hence find the length of each side.

3) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number her friend has.
Form an equation, letting *n* be the number of photographs one girl had at the **beginning**.
Hence find how many each has **now**.

## Chapter 3: SIMULTANEOUS EQUATIONS

**Example**: Solve $2x + 5y = 16$ ①

$3x - 4y = 1$ ②

**Solution**: We can get 20*y* in both equations if we multiply the equations by 4 and 5 respectively:

$8x + 20y = 64$ ③

$15x - 20y = 5$ ④

As the signs in front of 20*y* are different, we can eliminate the *y* terms from the equations by adding:

$23x = 69$ ③+④

i.e. $x = 3$

Substituting this into equation ① gives:

$6 + 5y = 16$

$5y = 10$

So... $y = 2$

The solution is $x = 3$, $y = 2$.

**Exercise A**:

Solve the pairs of simultaneous equations in the following questions:

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$

4) $9x - 2y = 25$

$2x + 3y = -6$ $\quad$ $4x - 5y = 7$

5) $4a + 3b = 22$, $5a - 4b = 43$

6) $3p + 3q = 15$, $2p + 5q = 14$

## Chapter 4: FACTORISING

**Example 1**: Factorise $12x - 30$

**Solution**: 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket. $12x - 30 = 6(2x - 5)$

**Example 2**: Factorise $6x^2 - 2xy$

**Solution**: 2 is a common factor to both 6 and 2. Both terms also contain an $x$. So we factorise by taking $2x$ outside a bracket. $6x^2 - 2xy = 2x(3x - y)$

**Exercise A**

Factorise

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

**Example 1**: Factorise $x^2 - 9x - 10$.

**Solution**: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1. Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

**Exercise B**

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2y$

14) $8(x+1)^2 - 2(x+1) - 10$

## Chapter 5: CHANGING THE SUBJECT OF A FORMULA

**Example 1**: Make $x$ the subject of the formula $y = 4x + 3$.

**Solution**: $y = 4x + 3$

Subtract 3 from both sides: $y - 3 = 4x$

Divide both sides by 4; $\dfrac{y-3}{4} = x$

So $x = \frac{y-3}{4}$ is the same equation but with *x* the subject.

**Example 2**: Make *x* the subject of $y = 2 - 5x$

**Solution**: Notice that in this formula the *x* term is negative.

| | | |
|---|---|---|
| | $y = 2 - 5x$ | |
| Add 5*x* to both sides | $y + 5x = 2$ | (the *x* term is now positive) |
| Subtract *y* from both sides | $5x = 2 - y$ | |
| Divide both sides by 5 | $x = \frac{2-y}{5}$ | |

**Exercise A**

Make *x* the subject of each of these formulae:

1) $y = 7x - 1$ 2) $y = \frac{x+5}{4}$ 3) $4y = \frac{x}{3} - 2$ 4) $y = \frac{4(3x-5)}{9}$

**Example 4**: Make *x* the subject of $x^2 + y^2 = w^2$

| | | |
|---|---|---|
| **Solution**: | $x^2 + y^2 = w^2$ | |
| Subtract $y^2$ from both sides: | $x^2 = w^2 - y^2$ | (this isolates the term involving *x*) |
| Square root both sides: | $x = \pm\sqrt{w^2 - y^2}$ | |

Remember that you can have a positive or a negative square root.

**Exercise B:**

Make *t* the subject of each of the following

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

**Example 6**: Make *t* the subject of the formula $a - xt = b + yt$

**Solution**: $a - xt = b + yt$

Start by collecting all the t terms on the right hand side:

Add *xt* to both sides: $a = b + yt + xt$

Now put the terms without a *t* on the left hand side:

Subtract *b* from both sides: $a - b = yt + xt$

Factorise the RHS: $a - b = t(y + x)$

Divide by (*y* + *x*): $\frac{a-b}{y+x} = t$

**Exercise C**

Make $x$ the subject of these formulae:

1) $ax + 3 = bx + c$ 2) $3(x + a) = k(x - 2)$ 3) $y = \frac{2x+3}{5x-2}$ 4) $\frac{x}{a} = 1 + \frac{x}{b}$

## Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation has the form $ax^2 + bx + c = 0$.

There are two methods that are commonly used for solving quadratic equations: factorising & the quadratic formula.

**Method 1: Factorising**

Make sure that the equation is rearranged so that the right hand side is 0. It usually makes it easier if the coefficient of $x^2$ is positive.

**Example 1** : Solve $x^2 - 3x + 2 = 0$
Factorise $(x - 1)(x - 2) = 0$
So the solutions are $x = 1$ or $x = 2$

**Method 2: Using the formula**

Recall that the roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Example 2**: Solve the equation $2x^2 - 5 = 7 - 3x$

**Solution**: First we rearrange so that the right hand side is 0. We get $2x^2 + 3x - 12 = 0$
We can then tell that $a = 2$, $b = 3$ and $c = -12$.
Substituting these into the quadratic formula gives:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-12)}}{2 \times 2} = \frac{-3 \pm \sqrt{105}}{4}$$ (this is the *surd form* for the solutions)

**Exercise A**

1) Use factorisation to solve the following equations:

a) $x^2 + 3x + 2 = 0$ b) $x^2 - 3x - 4 = 0$ c) $x^2 = 15 - 2x$

2) Find the roots of the following equations:

a) $x^2 + 3x = 0$ b) $x^2 - 4x = 0$ c) $4 - x^2 = 0$

3) Solve the following equations either by factorising or by using the formula:

a) $6x^2 - 5x - 4 = 0$ b) $8x^2 - 24x + 10 = 0$

4) Use the formula to solve the following equations. Some of the equations can't be solved.

a) $x^2 + 7x + 9 = 0$ c) $4x^2 - x - 7 = 0$ e) $3x^2 + 4x + 4 = 0$

b) $6 + 3x = 8x^2$ d) $x^2 - 3x + 18 = 0$ f) $3x^2 = 13x - 16$

## Chapter 7: INDICES

**Basic rules of indices**

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$ $4a^3 \times 6a^2 = 24a^5$

2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$ $24d^7 \div 3d^2 = \frac{24d^7}{3d^2} = 8d^5$

3) $(a^m)^n = a^{mn}$ e.g. $\left(3^2\right)^5 = 3^{10}$

**Exercise A**

Simplify the following:

1) $b \times 5b^5 =$

2) $3c^2 \times 2c^5 =$

3) $b^2c \times bc^3 =$

4) $2n^6 \times (-6n^2) =$

5) $8n^8 \div 2n^3 =$

6) $d^{11} \div d^9 =$

7) $\left(a^3\right)^2 =$

8) $\left(-d^4\right)^3 =$

**More complex powers**

**Zero index:**
Recall from GCSE that $a^0 = 1$.

**Negative powers**

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

**Fractional powers:**
Fractional powers correspond to roots: $a^{1/2} = \sqrt{a}$ $a^{1/3} = \sqrt[3]{a}$ $a^{1/4} = \sqrt[4]{a}$

**Exercise B:**

Find the value of:

1) $4^{1/2}$

2) $27^{1/3}$

3) $\left(1/9\right)^{1/2}$

4) $5^{-2}$

5) $18^0$

6) $7^{-1}$

7) $27^{2/3}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-2/3}$

10) $(0.04)^{1/2}$

11) $\left(\frac{8}{27}\right)^{2/3}$

12) $\left(\frac{1}{16}\right)^{-3/2}$

Simplify each of the following:

13) $2a^{1/2} \times 3a^{5/2}$

15) $\left(x^2 y^4\right)^{1/2}$

14) $x^3 \times x^{-2}$